FACEx RECOGNITION USING PCA, LDA AND VARIOUS DISTANCE CLASSIFIERS

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Abstract: Face recognition has become a major field of interest these days. Face recognition algorithms are used in a wide range of applications such as security control, crime investigation, and entrance control in buildings, access control at automatic teller machines, passport verification, identifying the faces in a given databases. This paper discusses different steps involved in face recognition using Principal Component Analysis (PCA) and Linear Discriminant Analysis (LDA) and the different distance measures that can be used in face recognition.

Keywords: Face Recognition, Principal Component Analysis, Linear Discriminant Analysis, LDA, PCA, distance measures.

INTRODUCTION

Face recognition is biometric identification by scanning a person’s face and matching it against a library of known faces. Face recognition is defined as the identification of a person from an image of their face. The success of any recognition method depends heavily on the particular choice of features used by the classifier. A good feature extractor is claimed to select features which are not sensitive to arbitrary environmental variations such as orientation and illumination [1]. Recognition systems can operate in well-controlled or uncontrolled environments. Image Recognition in well-controlled environments, where the imaging conditions of the trainee as well as the probe images are fixed, is relatively mature field of research [2]. Research in uncontrolled environments is much less mature and the results from well-controlled environments cannot be assumed to hold in uncontrolled environments. Recognition in controlled environments can be time and cost intensive and can be impractical to use in real world use [2]. As part of this research, main emphasis is on the assessment of suitability of image recognition systems in uncontrolled environment and their ability to use in real-world.

A wide variety of recognition methods for image recognition (fig. 1), especially for face image recognition are reported in the literature [3]. In this survey various methods for image recognition are categorized as Holistic methods [4-6], Feature-based methods [7-9], Hybrid methods [10]. Holistic methods use the whole face region as the raw input to a recognition system [3]. One of the most widely used representations of the face region is eigenfaces, which are based on principal component analysis and use a nearest neighbour classifier [4]. Fisherfaces which use linear/Fisher discriminant analysis (FLD/LDA) for best discriminating the face images of same class [5-6]. In Feature-based (structural) matching methods, local features such as the eyes, nose and mouth are first extracted and their locations and local statistics (geometric and/or appearance) are fed into a structural classifier [3]. Earlier methods belong to the category of structural matching methods, use the distances and angles between eye corners, mouth extreme, nostrils, and chin top [7]. Hidden Markov Model (HMM) based methods use strips of pixels that cover the forehead, eye, nose, mouth, and chin [8]. The Elastic Bunch Graph Matching (EBGM) algorithm stores spectral information about the neighbourhoods of facial features by convolving these areas with Gabor wavelets (masks) [9]. The Hybrid methods, just as the human perception system uses both local features and the whole face region to recognize a face. One can argue that these methods could potentially offer the better of the two types of methods [3].

One of the methods of this category is based on recent advances in component-based detection/recognition and 3D morphable models. The basic idea of component-based methods is to decompose a face into a set of facial components such as mouth and eyes that are interconnected by a flexible geometrical model. The 3D morphable face model is applied to generate arbitrary synthetic images under varying pose and illumination. Only three face images (frontal, semi-profile, profile) of a person are needed to

![Figure 1: Different techniques of Image Recognition](image-url)
compute the 3D face model [10]. The techniques used in this paper are based on holistic approaches.

**VARIOUS STEPS IN FACE RECOGNITION**

![Diagram of face recognition process]

**Image Acquisition:**
The method for acquiring face images depends upon the underlying application. For instance, surveillance applications may best be served by capturing face images by means of a video camera while image database investigations may require static intensity images taken by a standard camera. Some other applications, such as access to top security domains, may even necessitate the forgoing of the nonintrusive quality of face recognition by requiring the user to stand in front of a 3D scanner or an infra-red sensor [21].

**Discrete Wavelet Transform (DWT):**
The wavelet transform concentrates the energy of the image signals into a small number of wavelet coefficients. It has good time-frequency localization property [11]. The fundamental idea behind wavelets is to analyse signal according to scale. It was developed as an alternative to the short time Fourier to overcome problems related to its frequency and time resolution properties [12]. Wavelet transform decomposes a signal into a set of basic functions. These basic functions are obtained from a mother wavelet by translation and dilation.

\[ \Psi_{a,b}(t) = \frac{1}{\sqrt{a}} \, \Psi \left( \frac{t-b}{a} \right) \]  

Where \(a\) and \(b\) are both real numbers which quantify the scaling and translation operations respectively [13]. The advantage of DWT over DFT and DCT is that DWT performs a multi-resolution analysis of signal with localization in both time and frequency. Also, functions with discontinuities and with sharp spikes require fewer wavelet basis vectors in the wavelet domain than sine-cosine basis vectors to achieve a comparable approximation [14].

**Feature Extraction using PCA or LDA:**

**Principal Component Analysis (PCA):**
PCA is also known as Karhunen Loève projection. PCA calculates the Eigen vectors of the covariance matrix, and projects the original data onto a lower dimensional feature space, which is defined by Eigen vectors with large Eigen values. PCA has been used in face representation and recognition where the Eigen vectors calculated are referred to as Eigen faces. In gel images, even more than in human faces, the dimensionality of the original data is vast compared to the size of the dataset, suggesting PCA as a useful first step in analysis. There are many approaches to face recognition ranging from the Principal Component Analysis (PCA) approach (also known as Eigen faces) Prediction through feature matching. The idea of feature selection and point matching has been used to track human motion. Eigen faces have been used to track human faces.
They use a principal component analysis approach to store a set of known patterns in a compact subspace representation of the image space, where the subspace is spanned by the Eigen vectors of the training image set.

PCA is a useful statistical technique that has found application in fields such as face recognition and image compression, and is a common technique for finding patterns in data of high dimension. The basic goal is to implement a simple face recognition system, based on well-studied and well-understood methods. One can choose to go into depth of one and only one of those methods. The method to be implemented is the PCA (Principal Component Analysis). It is one of the more successful techniques of face recognition and easy to understand and describe using mathematics. This method involves using Eigen faces.

The first step is to produce a feature detector (dimension reduction). Principal Components Analysis (PCA) was chosen because it is the most efficient technique, of dimension reduction, in terms of data compression. This allows the high dimension data, the images, to be represented by lower dimension data and so hopefully reducing the complexity of grouping the images [19]. PCA aims to maximize between-class data separation [17]. It works by finding a new coordinate system for a set of data, where the axes (or principal components) are ordered by the variance contained within the training data [14]. A brief view of PCA is given below [4].

Step1: A set of M images (I₁, I₂, I₃...Iₘ) with size N×N can be represented by column or row vector of size N².
Step2: The average (μ) of the training set image is defined by

\[ \mu = \frac{1}{M} \sum_{n=1}^{M} I_n \]  

Step3: Each trainee image differs from the average image by vector (Φ)

\[ \Phi_i = I_i - \mu \]  

Step4: Total Scatter Matrix or Covariance Matrix is calculated from Φ as follows:

\[ C = \frac{1}{M} \sum_{n=1}^{M} \Phi_n \Phi_n^T \]  

\[ = AA^T \] , where A=[Φ₁,Φ₂,Φ₃...Φₘ]

Step5: Calculate the eigenvalues λₖ and eigenvectors u₀ of the covariance matrix C.

Step6: To classify an image, it can be projected into this feature space. Calculate the vectors of weights

\[ \Omega^T = [\omega_1, \omega_2... \omega_M], \]  

Where,

\[ \omega_k = u_k^T(I - \mu), \quad k = 1,2..., M \]  

Where \( u^T \) represents the total eigenvectors, but the ones with greater values.

Benefits of PCA:

(a). There is no data redundancy as components are orthogonal [19].
(b). With the help of PCA, complexity of grouping the images can be reduced [19].

(c). Smaller representation of database because we only store the trainee images in the form of their projections on the reduced basis [20].
(d). Noise is reduced because we choose the maximum variation basis and hence features like background with small variation are automatically ignored [20].

Limitation of PCA:

PCA treats inner-class and out-class equally and therefore it is sensitive to illumination changes [1].

**Equation:** \( S_{pca} = S_{w} + S_{b} \)

More formally, \( W_{opt} \) for Subspace LDA is givenby

\[ W_{opt}^T = W_{fld}^T \cdot W_{pca}^{-1} \]  

Where,

\[ W_{pca} = \text{arg max}_w |W^T S_p W| \]  

\[ W_{fld} = \text{arg max}_w |W^T W_{pca} S_{pca}^{-1} W| \]  

Linear Discriminant Analysis (LDA):

Linear Discriminant is a “classical” technique in pattern recognition, where it is used to find a linear combination of features which characterize or separate two or more classes of objects or events. The resulting combination may be used as a linear classifier or, more commonly, for dimensionality reduction before it can be classified [26].

In computerized face recognition, each face is represented by a large number of pixel values. Linear discriminant analysis is primarily used here to reduce the number of features to a more manageable number before classification. Each of the new dimensions is a linear combination of pixel values, which form a template. The linear combinations obtained using Fisher's linear discriminant are called Fisher faces, while those obtained using the related principal component analysis are called eigenfaces [26].

Linear Discriminant Analysis easily handles the case where the within-class frequencies are unequal and their performance has been examined on randomly generated test data. This method maximizes the ratio of within-class variance to the within-class variance in any particular data set thereby guaranteeing maximal separability. Data sets can be transformed and test vectors can be classified in the transformed space by two different approaches.

a. **Class-dependent transformation:** This type of approach involves maximizing the ratio of between class variance to within class variance. The main objective is to maximize this ratio so that adequate class separability is obtained. The class-specific type approach involves using two optimizing criteria for transforming the data sets independently.

b. **Class-independent transformation:** This approach involves maximizing the ratio of overall variance to within class variance. This approach uses only one optimizing criterion to transform the data sets and hence all data points irrespective of their class identity are transformed using this transform. In this type of LDA, each class is considered as a separate class against all other classes [26].
Difference between PCA and LDA:

The prime difference between LDA and PCA is that LDA deals directly with discrimination between classes, whereas the PCA deals with the data in its entirety for the principal components analysis without paying any particular attention to the underlying class structure [27]. In PCA, the shape and orientation of the original data set changes when transformed to a different space whereas LDA does not change the location but only tries to provide more class separability and draw a decision region between the given classes. The goal of the Linear Discriminant Analysis (LDA) is to find an efficient way to represent the face vector space. PCA constructs the face space using the whole face training data as a whole, and not using the face class information. On the other hand, LDA uses class specific information which best discriminates among classes. LDA produces an optimal linear discriminant function which maps the input into the classification space in which the class identification of this sample is decided based on some metric such as Euclidean distance. LDA takes into account the different variables of an object and works out which group the object most likely belongs to [26].

In Figure 6, there are two different classes represented by two different Gaussian-like distributions. However, only two samples per class are supplied to the PCA or LDA. In this conceptual depiction, the classification result of the PCA procedure (using only the first eigenvector) is more desirable than the result of the LDA. $D_{PCA}$ and $D_{LDA}$ represent the decision thresholds obtained by using nearest-neighbour classification [27].

One characteristic of both PCA and LDA is that they produce spatially global feature vectors. In other words, the basis vectors produced by PCA and LDA are non-zero for almost all dimensions, implying that a change to a single input pixel will alter every dimension of its subspace projection. At one level, PCA and LDA are very different: LDA is a supervised learning technique that relies on class labels, whereas PCA is an unsupervised technique [28].

LDA and PCA optimize the transformation $T$ with different intentions. LDA optimizes $T$ by maximizing the ration of between-class variation and within-class variation. PCA obtains $T$ by searching for the directions that have largest variations. Therefore LDA and PCA project parameter vectors along different directions. Figure 7 shows the difference between the projecting directions of LDA and PCA when projecting the parameter vectors from a two-dimensional parametric space onto a one-dimensional feature space [29].

The comparison table between PCA and LDA is given in Figure 8:

<table>
<thead>
<tr>
<th>Features</th>
<th>Principal Component Analysis</th>
<th>Linear Discriminant Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discrimination between classes</td>
<td>PCA deals with the data in its entirety for the principal components analysis without paying any particular attention to the underlying class structure.</td>
<td>LDA deals directly with discrimination between classes.</td>
</tr>
<tr>
<td>Supervised learning technique</td>
<td>PCA is an unsupervised technique.</td>
<td>LDA is a supervised learning technique that relies on class labels.</td>
</tr>
<tr>
<td>Focus</td>
<td>PCA searches for the directions that have largest variations.</td>
<td>LDA maximizes the ration of between-class variation and within-class variation.</td>
</tr>
<tr>
<td>Directions of maximum discrimination</td>
<td>The directions of maximum variance are not necessarily the directions of the maximum discrimination since there is no attempt to use the class information such as the between-class scatter and within-class scatter.</td>
<td>LDA is guaranteed to find the optimal discriminant directions when the class densities are Gaussian with the same covariance matrix for all the classes.</td>
</tr>
<tr>
<td>Well distributed classes in small datasets</td>
<td>PCA is less superior to LDA.</td>
<td>LDA is superior to PCA.</td>
</tr>
<tr>
<td>Computations for large datasets</td>
<td>PCA requires fewer computations.</td>
<td>LDA requires significantly more computation than PCA for large datasets.</td>
</tr>
<tr>
<td>Applications</td>
<td>Application of PCA in the prominent field of criminal investigation is beneficial.</td>
<td>Linear Discriminant Analysis for data classification is applied to classification problem in speech recognition.</td>
</tr>
</tbody>
</table>

Distance Measures:

Various distance measures can be used as similarity measure to compare the feature vector of test image with that of trainee images. All the trainees as well as the test image are projected to the feature space of training dataset. Distances between the projected test image and the projection of all centred trainee images are calculated. Test image is...
supposed to have minimum distance with its corresponding equivalent image in the training dataset.

**Types of Distance Measures:**
The various types of distance measures that can be used in face recognition are explained below [22]:

**City Block distance:**
The sum of absolute differences between two vectors is called the L1 distance, or city-block distance. This is a true distance function since it obeys the triangle inequality. reason why it is called the city-block distance, and also as the Manhattan distance or taxicab distance is that going from a point A to a point B is achieved by walking ‘around the block’, compared to the Euclidean ‘straight line’ distance[23].

\[ d(x, y) = \sum_{i=1}^{k} |x_i - y_i| \]

**Euclidean Distance:**
Euclidean distance, or simply 'distance', examines the root of square differences between the coordinates of a pair of objects. This is most generally known as the Pythagorean Theorem. For testing we used the Euclidean distance classifier, for calculating the minimum distance between the test image and image to be recognized from the database. If the distance is small, we say the images are similar and we can decide which the most similar image in the database is [25]. Euclidean distance is one of the simplest and faster classifier as compared to other classifiers. Euclidean distance is defined as the straight-line distance between two points. Minimum Euclidean distance classifier is optimum for normally distributed classes.

\[ d_2(x, y) = \sqrt{\sum_{i=1}^{k} (x_i - y_i)^2} \]

**The Squared Euclidean Distance:**
Without the square roots, we obtain the Squared Euclidean distance Classifier as follows:

\[ d(x, y) = \sum_{i=1}^{k} (x_i - y_i)^2 \]

**Angle:**
Negative Angle between Image Vectors is represented as follows:

\[ d(x, y) = -\frac{x \cdot y}{\|x\| \|y\|} = -\frac{\sum_{i=1}^{k} x_i y_i}{\sqrt{\sum_{i=1}^{k} (x_i)^2} \sqrt{\sum_{i=1}^{k} (y_i)^2}} \]

**Mahalanobis Distance:**
Mahalanobis distance is a distance measure introduced by P. C. Mahalanobis in 1936. It is based on correlations between variables by which different patterns can be identified and analyzed. It gauges similarity of an unknown sample set to a known one. It differs from Euclidean distance in that it takes into account the correlations of the data set and is scale-invariant [24].

\[ d(x, y) = -\sum_{i=1}^{k} \frac{1}{\lambda_i} x_i y_i \]

Where \( \lambda_i \) is the ith eigenvalue corresponding to the ith eigenvector

**Combining distance measures:**
Rather than using a single distance classifier for finding the distance between images, some combination of the above given standard distance measures (City Block, Euclidean, angle and Mahalanobis) might outperform the individual distance measures. The simplest mechanism for combining distance measures is to add them.

**Rotation of test image:**
The recognition accuracy of the face recognition system can be improved by rotating the test image at different angles such as 90, 180, 270.

**CONCLUSION**
The major steps involved in face recognition are:- image acquisition, applying DWT, feature extraction using PCA or LDA, selecting distance measure and finally rotating the image at different angles if match is not found. PCA technique is unsupervised learning technique that is best suited for databases having images without class labels, whereas LDA is supervised learning technique that relies on class labels and is well suited for distributed classes in small datasets. Different distance measures or classifiers may be used for finding the distance between trainee image and database images such as Euclidean distance, city-block distance, angle distance classifier, mahalanobis distance etc. Rather than using a single distance classifier for finding the distance between images, some combination of the standard distance measures (City Block, Euclidean, angle and Mahalanobis) might outperform the individual distance measures. The simplest mechanism for combining distance measures is to add them.

**REFERENCES**


Short Bio Data for the Author

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